Math 214 – Introductory Statistics **Summer 2008**
6-17-08 Class Notes

Sections 5.2, 5.3
5.2: 7-17 odd
5.3: 1-17 odd

Probability Distributions and Expectation

Definition: A ***random variable*** is a variable whose values are determined by chance. Random variables can be ***discrete*** (if the possible values can be *counted*) or ***continuous*** (if the possible values are *measured*).

Example 1: List three examples of discrete random variables and three examples of continuous random variables.

Discrete: Inventory, population, survey
Continuous: Temperature, time, weight

Example 2: Suppose we flip a coin 4 times. We have the following sample space:

Outcomes
H H H H
H H H T
H H T H
H T H H
T H H H
H H T T
H T H T
T H H T
H T T H
T H T H
T T H H
H T T T
T H T T
T T H T
T T T H
T T T T

Now, let's suppose we want to count the tails that occur. Let's add a column to our table:

Outcomes	Tails
H H H H	0
H H H T	1
H H T H	1
H T H H	1
T H H H	1
H H T T	2
H T H T	2
T H H T	2
H T T H	2
T H T H	2
T T H H	2
H T T T	3
T H T T	3
T T H T	3
T T T H	3
T T T T	4

So in our 16 outcomes, 1 yielded no tails, 4 yielded one tail, 6 yielded two tails, 4 yielded three tails, and 1 yielded four tails. The number of tails is a discrete random variable. We can summarize this fact with a table:

Outcome	Probability
0 Tails	$\frac{1}{16}$
1 Tail	$\frac{4}{16}$
2 Tails	$\frac{6}{16}$
3 Tails	$\frac{4}{16}$
4 Tails	$\frac{1}{16}$

This is called a ***discrete probability distribution***. In general, this looks like this:

Outcome	Probability
x_1	p_1
x_2	p_2
x_3	p_3
$\vdots$	$\vdots$
x_n	p_n

There are some properties of discrete probability distributions that should be clear:

(1) The probability of each outcome in the sample space must between 0 and 1 (inclusive). That is, $0 \le p_i \le 1$ for all i.

(2) The sum of all the probabilities must equal 1. That is, $\sum p_i = 1$.

Mean and Variance of a Discrete Random Variable

Definition: The ***mean*** of a discrete random variable (a.k.a the ***expected value)*** is given by $\mu = E(X) = \sum (x \cdot p)$. The variance is given by $\sigma^2 = \sum (x^2 \cdot p) - \mu^2$. As before, the ***standard deviation*** is the square root of the variance $\sigma = \sqrt{\sigma^2}$.

Example 3: Compute the mean, variance, and standard deviation of the coin-flipping example above.

Recall, we had the following table (with a few new columns):

x	p	$x \cdot p$	x^2	$x^2 \cdot p$
0	$1/16$	$0/16$	0	$0/16$
1	$4/16$	$4/16$	1	$4/16$
2	$6/16$	$12/16$	4	$24/16$
3	$4/16$	$12/16$	9	$36/16$
4	$1/16$	$4/16$	16	$16/16$
		$32/16$		$80/16$

So $\mu = \frac{32}{16} = 2$, $\sigma^2 = \frac{80}{16} - 4 = \frac{16}{16} = 1$, and $\sigma = \sqrt{1} = 1$.

Notice what the mean (or expected value) represents. Since $\mu = 2$, on average we should get two tails on four flips of a coin. Hence, "expected value."

Example 4: Suppose we roll two fair 6-sided dice and calculate the sum. Find the mean.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

So our probability distribution is:

x	p
2	$1/36$
3	$2/36$
4	$3/36$
5	$4/36$
6	$5/36$
7	$6/36$
8	$5/36$
9	$4/36$
10	$3/36$
11	$2/36$
12	$1/36$

So the mean sum is:

$$\mu = \sum x \cdot p = \left(2 \cdot \tfrac{1}{36}\right) + \left(3 \cdot \tfrac{2}{36}\right) + \cdots + \left(11 \cdot \tfrac{2}{36}\right) + \left(12 \cdot \tfrac{1}{36}\right) = \tfrac{252}{36} = 7$$

Example 5: The number of suits sold per day at a retail store is shown in the given probability distribution. Find the mean, variance, and standard deviation of the distribution. If the manager of the store wants to be sure he has enough suits for the next 5 days, how many should he have?

x	p
19	0.2
20	0.2
21	0.3
22	0.2
23	0.1

x	p	$x \cdot p$	x^2	$x^2 \cdot p$
19	0.2	3.8	361	72.2
20	0.2	4.0	400	80.0
21	0.3	6.3	441	132.3
22	0.2	4.4	484	96.8
23	0.1	2.3	529	52.9
		20.8		434.2

So $\mu = 20.8$, $\sigma^2 = 434.2 - (20.8)^2 = 1.6$, and $\sigma = \sqrt{1.6} = 1.3$. Since the manager "expects" to sell 20.8 suits a day, he should stock 104 suits for the next 5 days.

Example 6: If a player rolls two dice and gets a sum of 2 or 12, she wins \$20. If she gets a 7, she wins \$5. Otherwise, she wins nothing. It costs \$3 to play. Find and interpret her expected value.

x	p
$+\$17$	$\frac{2}{36}$
$+\$2$	$\frac{6}{36}$
$-\$3$	$\frac{28}{36}$

So $\mu = \left(17 \cdot \frac{2}{36}\right) + \left(2 \cdot \frac{6}{36}\right) + \left(-3 \cdot \frac{28}{36}\right) = -\frac{38}{36} = -1.06$. On average, she will lose \$1 every time she plays.